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Abstract

Hong Kong's currency is pegged to the US dollar in a currency board arrangement. In autumn 2003, the Hong Kong dollar appreciated from close to 7.80 per US dollar to 7.70, as investors feared that the currency board would be abandoned. In the wake of this appreciation, the monetary authorities revamped the one-sided currency board mechanism into a symmetric two-sided system with a narrow exchange rate band. This paper reviews the characteristics of the new currency board arrangement and embeds a theoretical soft edge target zone model typifying many intermediate regimes, to explain the notable achievement of speculative peace and credibility since May 2005.

Keywords: Currency Board Arrangement, Target Zone Model, Credibility, Hong Kong
JEL-Classification: C61, E42, F31, F32

1. Introduction

Hong Kong´s currency board stands out among such arrangements around the world as the one with the longest history.[1] Hong Kong´s currency board was established in 1983 and the Hong Kong dollar (HKD) was pegged to the US dollar (USD) at 7.80 to 1. In September 1998, the rate was changed to 7.75 to 1. The exchange rate moved gradually from 7.75 back to 7.80 between April 1999 and August 2000. The HKD is freely convertible and the Hong Kong Monetary Authority (HKMA) is responsible for the peg.

In the present currency board setup, the note-issuing banks deliver to the HKMA an equivalent amount of US dollars (USD) as backing for their HKD note issues. The HKMA in turn issues certificates of indebtedness denominated in HKD. Similarly, the banks can obtain USD from the HKMA by delivering local currency at the fixed exchange rate. Other banks wishing to exchange local currency notes can also purchase (or sell) them against equivalent amounts of USD at the fixed exchange rate.[2]

Contrary to actively managed fixed exchange rate systems, a currency board system is a passive "hard-fixed" peg system. Nonetheless, the HKMA does at times intervene in the foreign exchange market to defend the peg. In addition, the HKMA influences interbank liquidity and thus short-term inter-bank interest rates. Hong Kong´s currency board has survived a number of booms and busts, including a massive speculative attack during Asia´s financial crisis of 1997-98. Given the speculative outflow of USD, the HKMA sold large amounts of USD in October 1998 to defend the peg. Furthermore, the HKMA pursued a defensive interest rate strategy which was partly responsible for bringing about a severe recession.[3]

In light of the fact that exchange rate regimes have been at the centre of academic debate and have been a major concern for policymakers in recent years, the paper proceeds as follows. Section 2 sheds light on the dynamics of the exchange rate in the period 2001 - 2009 and reviews institutional features of Hong Kong´s currency board. Section 3 presents the model and its solution. Analytically, the modelling approach lies at the crossroads of the literature on exchange rate target zones originated by Krugman (1991) and the literature on currency boards. With the aim of parsimony in mind, but also wanting to ensure a fair degree of reality, we extend and generalize the standard

[1] Locally, Hong Kong´s currency board system is known as the "linked exchange rate system". Initially, the currency board was adopted as an emergency measure to prevent the HKD from collapsing during a political row between China and the United Kingdom in 1983 over the future of the colony.

[2] An in-depth discussion of Hong Kong´s currency board, including documentations on the technical details is available at http://www.info.gov.hk/hkma/eng/currency/link_ex/index.htm. For a perceptive and thorough discussion, see Latter (2007). Most economic policy contributions and discussions on currency board arrangements are based on informal analysis devoid of formal economic models. An exception is Chan and Chen (2003).

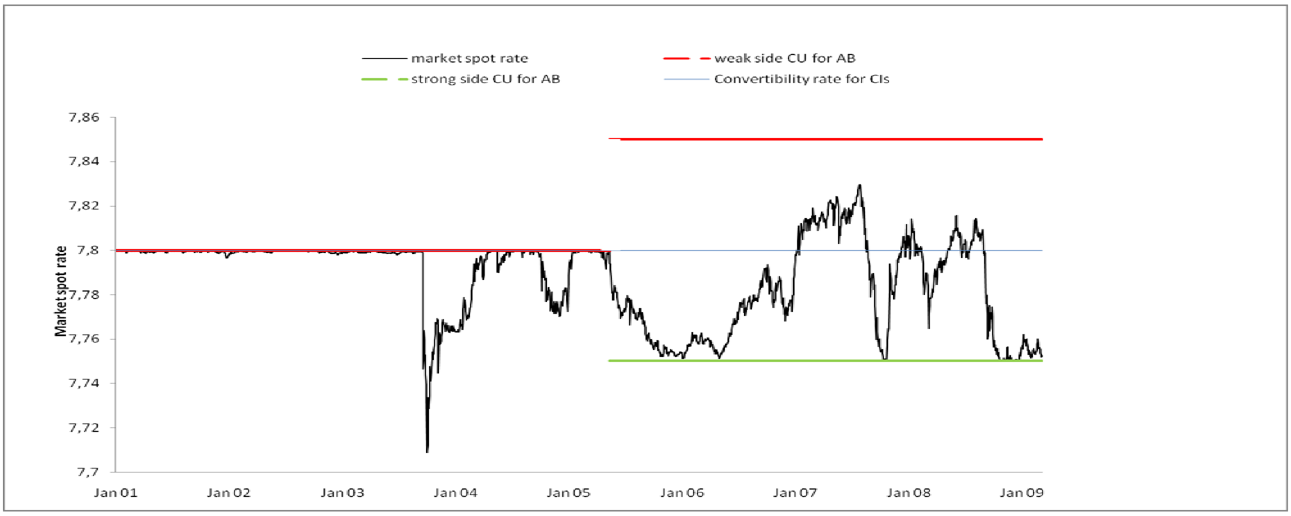
[3] The result of this surprise move was that interbank interest rates jumped and the overnight rate touched 280 percent. This successfully stemmed the speculative outflow of USD. Overnight rates dropped back to about 5 percent within a few days.

target zone model by introducing perceived uncertainty about the edges following the policy change in May 2005. Section 4 concludes and draws some policy implications.

2. Institutional Features of Hong Kong´s Currency Board, 2001 - 2009

Despite the presumed rigidity of the currency board system, the convergence between the exchange rate in the interbank market and the fixed rate for currency did not happen in practice. Initially, the HKMA therefore introduced a wider exchange rate commitment for reserve banks (weak-side Convertibility Undertaking – CU). This was a weak-side commitment in that the HKMA was ready to buy unlimited amounts of HKD for USD to prevent a weakening of the currency.[4] This weak-side commitment is shown by the red line in Figure 1.

Figure 1: Daily HKD Spot Exchange Rate Against USD and Exchange Rate Band



Source: Bloomberg

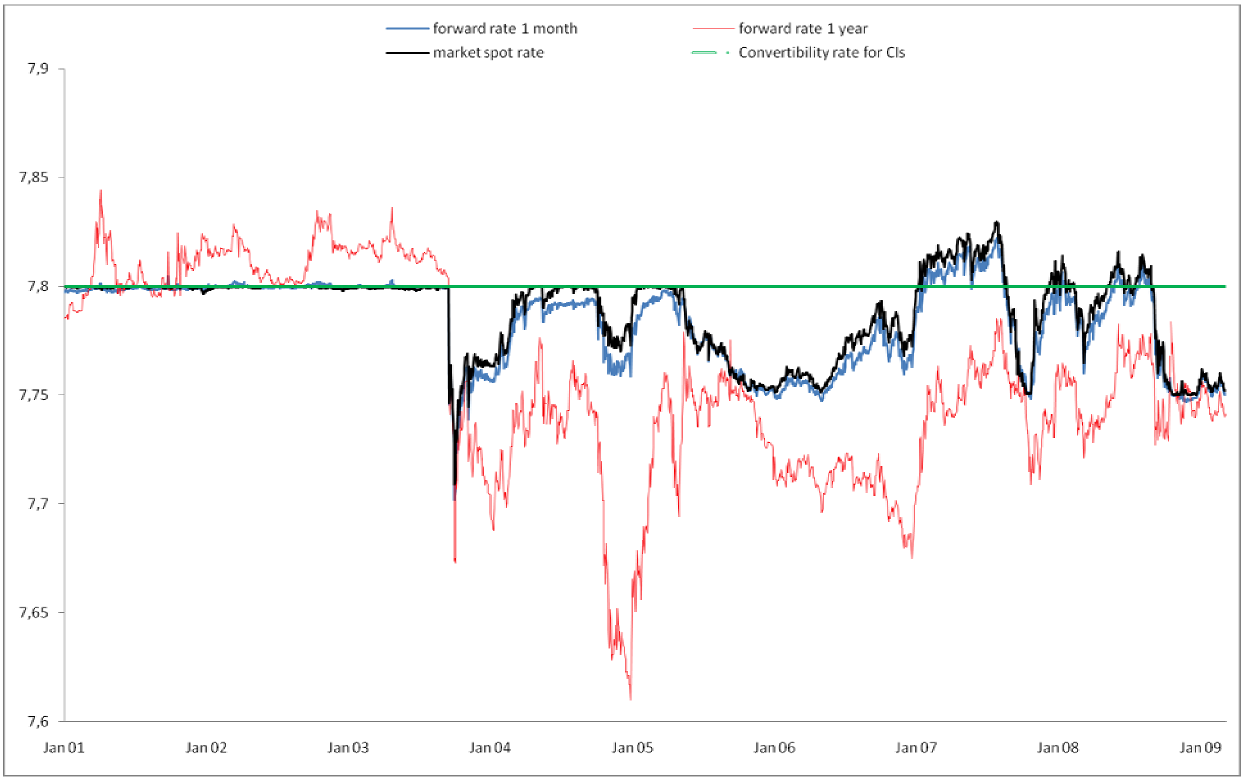
Since late 2003, speculative pressure for a revaluation of the Chinese Renminbi has grown, with the result of large speculative inflows. The HKD appreciated from 7.80 to 7.70 in autumn 2003, fuelling speculation that the currency board link to the USD might be abandoned.[5] The experience shows that a currency board is not a form of magic protection against speculation. Lastly, the link was defended with a combination of market interventions, including direct foreign exchange operations and manipulation of liquidity and interest rates. Over and above these actions, the currency board

[4] In contrast, no strong-side boundary was introduced, i.e. the previous system was asymmetric. While no formal strong-side intervention point was introduced, the Subcommittee on Currency Board Operations already considered the options in this area in meetings in October 1999 and July 2000 and "agreed that there would be scope to review the arrangement again, should the need arise" [HKMA (2000)]. For a review of the performance of the currency board arrangement after May 2005, see Genberg et al. (2007).

[5] For the first time, this added a ceiling to the floor by which HKMA had traditionally managed the currency, in a move to discourage investors from using the HKD to speculate on RMB appreciation.

arrangement was again altered on 18 May 2005 when finally a narrow symmetric target zone of ±0.6 percent was introduced with a strong-side Convertibility Undertaking (the green line in Figure 1) at HKD 7.75/USD. For the first time, this added a ceiling to the floor by which it had traditionally managed the currency, in a move to discourage investors from using the HKD to speculate on a RMB appreciation. At the same time, the weak-side CU was changed from HKD 7.80/USD to HKD 7.85/USD. These "refinements" were intended to anchor market expectations and promote a smooth functioning of the currency board arrangement. Figure 1 shows that the HKD spot rate stayed close to the strong-side CU most of the time after May 2005. The validity of the current arrangement has not been called into question by the current financial crisis.

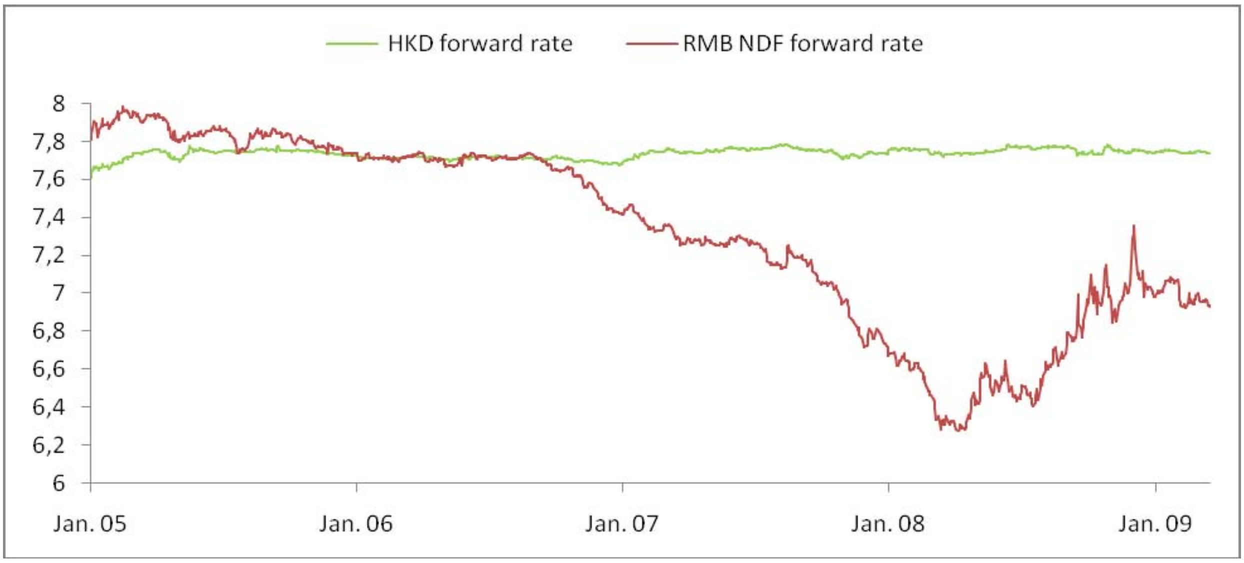
Figure 2: Daily Movements in HKD Spot Rate and Forward Rates Against USD



Source: Bloomberg

Figure 2 illustrates the credibility record of the Hong Kong experiment implemented in 2005. Time series for the (on-shore) 1-month and 1-year forward exchange rates of the HKD against the USD are shown in Figure 2. This is known as Svensson´s (1992, 1994) 100% credibility test. It rests on investigating whether the forward rate lies within the band (credibility holds) or whether the band is violated (credibility doesn´t hold). The chart indicates that prior to autumn 2003 the forward market expected a depreciation of the HKD (HKD rates were constantly higher than the target rate), which never materialized. Since the end of 2003, the forward market has consistently expected – and missed - an appreciation of the HKD relative to the USD. The red line for the 1-year forward rate indicates that it was far outside the official band in January 2005 and only unhurriedly moved towards the Convertibility Zone afterwards. All in all, the 1-year forward rate of the HKD was consistently outside the convertibility zone between October 2005 and the beginning of 2007. This indicates that financial market participants were initially sceptical about the ability of the new strong-side CU to limit exchange rate fluctuations.[6]

Figure 3: 1-Year Forward Exchange Rates of HKD and RMB Against USD



Source: Bloomberg

How can we explain this development? Figure 3 spotlights developments in the 1-year forward exchange rate of the HKD versus the non-deliverable 1-year forward exchange rate (NDF) of the RMB from 2005 to 2009.[7] Until autumn 2006 the 1-year HKD and the appreciating 1-year NDF RMB forward rates were parallelly aligned, after which the co-movement disappeared.[8] The intermittent upward pressure on the HKD forward exchange rate was in part the result of strong

[6] If the peg is viewed as irrevocable, the forward premium should be eliminated entirely. As Figure 2 indicates, the forward premium varied significantly over time, reflecting major domestic and international events that impacted Hong Kong´s current and/or anticipated economic and financial conditions. This raises the question of the credibility of the target zone bands, which has been unduly neglected as an important component of the target zone literature. Schmukler and Servén (2002) provide a comprehensive characterisation of the currency premium in Hong Kong. Their estimates raise the question of whether Hong Kong´s currency board really has sufficient credibility to minimise exchange rate risk.

[7] As is the case with standard forward contracts, NDF exchange rates are fixed for a future date. Thus, the NDF contract is similar to a regular forward foreign exchange contract, except that at maturity the NDF is settled in another currency, typically the USD, because the domestic currency is subject to capital controls, and is therefore “non-deliverable”. In contrast to standard deliverable forwards, NDF´s are traded offshore, i.e. outside the jurisdiction of the authorities of the corresponding currency. Trading predominantly takes place in Hong Kong and Singapore and, to a lesser extent, in London and Tokyo. The offshore markets offer international investors an otherwise unavailable hedging tool against local currency exposure. An analysis of Asian NDF markets as a whole, together with a discussion of the basic institutional features of the renminbi NDF market, is provided by Fung et al. (2004), Ma et al. (2004) and Debelle et al. (2006). Ma et al. (2004) and Debelle et al. (2006) discuss the deepening of the Asian NDF markets in recent years.

[8] This point was also raised by Genberg and Hui (2009).

demand for HKD to buy shares on Hong Kong´s stock market. An important factor driving this demand was the expectation that the Chinese authorities would allow mainland citizens to invest directly in Hong Kong. Belatedly, this reform was deferred indefinitely.

The perceived credibility issues arising from the new arrangement are modelled in the next section of the paper. The resulting setup provides a variation of the standard hard edge target zone concept.

3. A Soft Edge Target Zone Model

Below we rigorously study the perceived credibility issues of an emerging band system in a formal model. Our procedure can be regarded as a generalisation of Krugman´s (1991) simple target zone model of exchange rates. In the standard model, the peg and the hard edge boundaries are credible, and market arbitrage mechanisms and interventions take place in whatever amounts are necessary to prevent the band from being violated. In contrast, our model develops the notion that the exchange rate band can be seen as a partially credible commitment device.

In the Hong Kong case, two potential risks may have emerged after May 2005. The first was uncertainty about whether the HKMA would honour the peg. For the time being, the Hong Kong authorities remain markedly committed to maintaining the existing currency board system.[9] Though Hong Kong´s economy has become more closely integrated with that of mainland China and it may eventually make sense to replace the USD with the renminbi (RMB) in the peg, this "endgame" is rather unlikely to happen any time soon, owing to China´s closed capital account. This makes it technically impossible for Hong Kong to peg its currency to the RMB. In addition, given the almost unanimous belief that the RMB will continue to appreciate over the medium term, it makes little sense for Hong Kong to fix the HKD to a currency that is expected to rise in value. If the RMB rises further, Hong Kong´s economy – particularly the city´s important export sector - will become more competitive in the region. Therefore the USD link is unlikely to be abandoned soon. The second perceived risk was that the newly introduced symmetric band might be widened in the future.[10] In the event of a new exchange rate band, the financial markets may not base their expectations on a blind faith in the working of the currency board mechanism and the will and commitment of the monetary authorities to defend the edges of the band. If the markets can decipher the potential

[9] At the end of 2008, Hong Kong´s backing ratio was 109,5%. The backing ratio is defined as the ratio between the market value of the backing portfolio and the value of the monetary base. This will make it (almost) feasible to defend the preset "Maginot lines" against market pressure. Data are available from the HKMA *Monthly Statistical Bulletin* (http://www.info.gov.hk/hkma/eng/public/index.htm).

[10] HKMA would not be the first central bank to do this. For example, in the European Exchange Rate Mechanism (ERM), the currencies were initially allowed to fluctuate no more than 2.25 percent above or below fixed bilateral rates. The UK joined in 1990 but was forced to leave in 1992 when sterling came under speculative pressure. Fluctuation bands were then widened to plus or minus 15 percent in 1993 to avoid defending the indefensible. Likewise, Labhard and Wyplosz (1996) have shown that several central banks in the ERM have announced wide exchange rate bands while implicitly targeting narrower ones inside. In other words, there was a discrepancy between announced (known) and implicit (unknown) bands.

fragility of the edges and do the backward induction, a target zone may lose its reputation and stabilising power. Next we concentrate on this second case and discuss the question of perceived soft edges in a theoretical framework.

Let us first briefly consider the basic Krugman (1991) framework with perfectly known and credible bands to gain a benchmark for future analysis.[11] The model starts from the log-linear asset pricing equation that expresses the log exchange rate, *s*, as the sum of the log of the fundamental, *f*, and its own expected change:

$$(1) \qquad s = f + \lambda E[ds]/dt ,$$

where $E[\cdot]$ denotes the rational expectations operator, $\lambda > 0$, and time subscripts are omitted for brevity. The factors affecting the exchange rate are the fundamentals and financial markets' expectations about the future movement of the exchange rate. It is assumed that the log of the fundamental follows an arithmetic Brownian motion without drift:

$$(2) \qquad df = \sigma dW ,$$

where σ is the risk parameter and *W* denotes a standard Brownian motion. Applying Itô's lemma to the expectations term yields

$$(3) \qquad s = f + \frac{\lambda}{2}\sigma^2 \frac{d^2 s}{df^2} .$$

Krugman (1991) specified the dynamics of f between two limits. The lower and upper band limits, $\bar{s}$ and $\underline{s}$, result from the intervention obligations within the target zone arrangement. This gives rise to a reflected or regulated Brownian motion.[12]

The particular solution of equation (1) is as simple as $s = f$, showing the unregulated exchange rate dynamics. The homogeneous solution, s^H, representing the changes in values from the intervention of the central bank is

[11] A number of papers have modified and relaxed the assumption of perfectly known bands. Klein (1992), for instance, presented a model in which the width of the band is unknown to the public. The dynamics of the exchange rate are driven not only by fundamentals but also by expectations with respect to bandwidth. As a consequence, the exchange rate will react more sensitively to fundamentals than in Krugman (1991). On the other hand, an intervention reveals the true edge of the band. This mechanism stresses the information role of interventions. Klein and Lewis (1993) extended the model to the case of stochastic intra-marginal interventions.

[12] Some papers have endogenised exchange rate policy by deriving the width of the band as a rational choice of an optimising central bank. For example, Cukierman et al. (1994) model the choice of bandwidth as a choice between flexibility in responding to external shocks and commitment to less devaluation and inflation.

(4) $$s^H = -A_1 e^{\beta f} + A_2 e^{-\beta f} ,$$

where $\beta = \sqrt{2/\lambda} / \sigma$ and A_1 and A_2 are positive coefficients. The exponential terms $A_1 e^{\beta f}$ and $A_2 e^{-\beta f}$ cause the bending of the exchange rate function and thus generate the target zone nonlinearities. Expressing the exchange rate as an explicit function of the fundamental we obtain the *S*-shaped exchange rate function

(5) $$s = f - A_1 e^{\beta f} + A_2 e^{-\beta f} ,$$

which is the general solution of the form for exchange rates. The band edges come from the two value-matching conditions:

(6) $$\overline{s} = \overline{f} - A_1 e^{\beta \overline{f}} + A_2 e^{-\beta \overline{f}} ,$$

(7) $$\underline{s} = \underline{f} - A_1 e^{\beta \underline{f}} + A_2 e^{-\beta \underline{f}} ,$$

where for $\overline{s}$ and $\underline{s}$ are strong and weaker band exchange rates and $\overline{f}$ and $\underline{f}$ are the levels of fundamentals as the edges of the bands are approached. The no-expected-arbitrage-profits conditions imply the following two smooth-pasting conditions, which require that the exchange rate function is flat at the edges:

(8) $$0 = 1 - A_1 \beta e^{\beta \overline{f}} - A_2 \beta e^{-\beta \overline{f}} ,$$

(9) $$0 = 1 - A_1 \beta e^{\beta \underline{f}} - A_2 \beta e^{-\beta \underline{f}} .$$

Equations (6) – (9) enable determination of the two constants of integration, A_1 and A_2, which completely solves the model.[13] After obtaining $\overline{f}$, $\underline{f}$, A_1 and A_2, a *S*-shaped curve can be obtained from equation (5).

[13] Advocates of models with endogenous realignment risks argue that a high risk of realignment can reverse the *S*-type effect that the target zone has on the exchange rate [Bertola and Caballero (1992), Bertola and Svensson (1993)].

In the original Krugman (1991) model, the reduced form relationship between exchange rates and fundamentals is driven by the perfectly known and credible bands. Conversely, what happens when the relationship is driven not by the band itself but by expectations regarding the band? How can we model perceived uncertainty of market participants as to the currency board arrangement and Hong Kong´s monetary authorities' determination to defend the newly introduced symmetric bands? To keep the story tractable, the soft edge band is assumed to consist of two components: the announced edges $\underline{s}$ and $\bar{s}$, and a normally distributed noise term.[14] In normal times, the noise term is small and the self-adjusting autopilot function of the currency board system is expected to behave properly, i.e. endogenous market forces stabilise the exchange rate without any need for intervention.[15] But in noisy times, when confidence is fragile, capital inflows may cause a crisis of confidence that sets off massive capital flows. In other words, we replace the perfectly known and credible symmetric band [$\bar{s}$, $\underline{s}$] by [$N\left(\bar{s},\sigma_s^2\right), N\left(\underline{s},\sigma_s^2\right)$] and we analyse how the uncertainty as to the band feeds back into the dynamics of the exchange rate. The variance σ_S^2 reflects the degree of confidence market participants have in the band given the economy´s current and expected fundamentals.

To keep the exchange rate within the symmetric band [$N\left(\bar{s},\sigma_s^2\right), N\left(\underline{s},\sigma_s^2\right)$], it is sufficient to confine the fundamental process to [$N\left(\bar{f},\sigma_s^2\right)$, $N\left(\underline{f},\sigma_s^2\right)$] at both ends. By taking expectations conditional on the information of $\bar{f}$, $\underline{f}$, and σ_s^2, we obtain the following perceived value-matching conditions for the soft edge model, as shown in the Appendix:

[14] Reality is more complicated, as usual. It is possible to assume more complicated sets of assumptions in our model, such as private sector expectations on capital inflow shocks and posterior updates based on realisations, but we believe that our simple approach is an adequate and tractable representation of the situation. Another alternative would be to use a distribution with fatter-than-normal tails, which would imply a more bimodal pattern. Yet we stay with our approach, given the disappointing empirical record of structural exchange rate models and that our approach does not require imposition of excessive structure.

[15] The stabilising effect of a currency board arrangement is entirely different from a target zone system. A currency board issues currency notes with 100 percent foreign reserves backing at a fixed exchange rate. This represents a strong commitment to economic discipline. Any holder of paper money therefore rests assured that she can exchange her notes for foreign currency at the fixed rate. Since the exchange rate of paper money is fixed, so too must be the exchange rate for bank deposits. Any rate differential leads to profitable cash arbitrage, which closes the gap. If the prices of the same product in two sub-markets differ from each other, one can buy the product in the lower-price sub-market and sell it at a higher price in the other, gaining a profit at zero risk. As many market participants would engage in similar arbitrage, the two prices should equalize, provided transaction costs are negligible. The second market arbitrage mechanism is interest arbitrage. For example, if there is speculation against the currency, funds will flow out of the economy and domestic interest rates will rise. This should reverse the outflow and stabilise the exchange rate. Both market arbitrage mechanisms can be classified as self-reversing market movements and represent the self-adjusting "autopilot" of a currency board arrangement. It is like the honeymoon effect in a fully credible target zone, but it is driven by arbitrage discouraging the exchange rate from straying outside of the band.

$$E\left[s\middle|\overline{s},\sigma_s^2\right]=E\left[f-A_1e^{\beta f}+A_2e^{-\beta f}\middle|\overline{f},\sigma_s^2\right]$$

(10) $$\Rightarrow \overline{s}=\overline{f}-A_1e^{\beta\left(\overline{f}+\beta\sigma_s^2/2\right)}+A_2e^{-\beta\left(\overline{f}-\beta\sigma_s^2/2\right)}$$

$$\Rightarrow \overline{s}=\overline{f}-\left(A_1e^{\beta\overline{f}}-A_2e^{-\beta\overline{f}}\right)e^{\beta^2\sigma_s^2/2},$$

$$E\left[s\middle|\underline{s},\sigma_s^2\right]=E\left[f-A_1e^{\beta f}+A_2e^{-\beta f}\middle|\underline{f},\sigma_s^2\right]$$

(11) $$\Rightarrow \underline{s}=\underline{f}-A_1e^{\beta\left(\underline{f}-\beta\sigma_s^2/2\right)}+A_2e^{-\beta\left(\underline{f}-\beta\sigma_s^2/2\right)}$$

$$\Rightarrow \underline{s}=\underline{f}-\left(A_1e^{\beta\underline{f}}-A_2e^{-\beta\underline{f}}\right)e^{\beta^2\sigma_s^2/2}.$$

The corresponding smooth-pasting conditions are

(12) $$0=1-A_1\beta e^{\beta\left(\overline{f}+\beta\sigma_s^2/2\right)}-A_2\beta e^{-\beta\left(\overline{f}-\beta\sigma_s^2/2\right)},$$

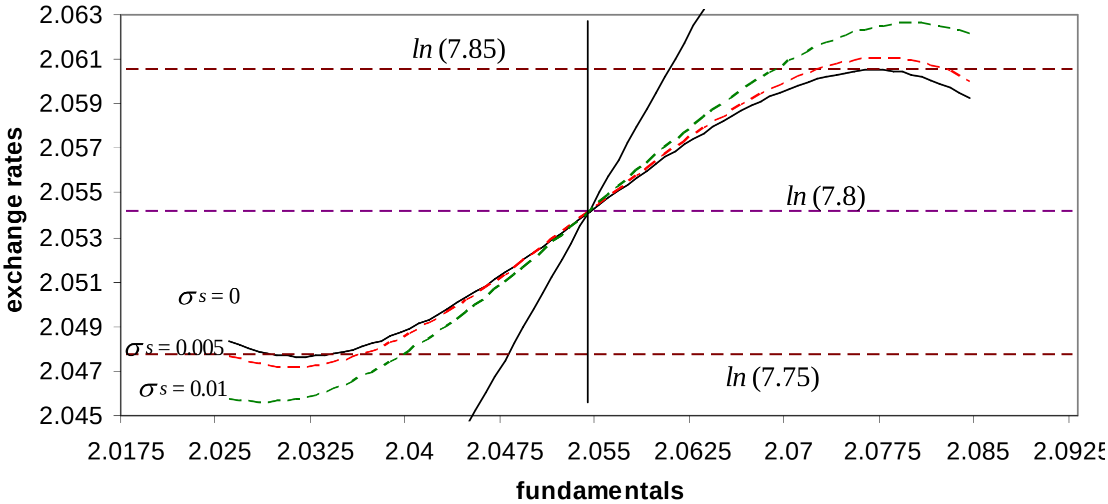
(13) $$0=1-A_1\beta e^{\beta\left(\underline{f}+\beta\sigma_s^2/2\right)}-A_2\beta e^{-\beta\left(\underline{f}-\beta\sigma_s^2/2\right)}.$$

Next we study the properties of this model. The key difference between the solution for a fully credible hard band and a perceived soft edge band is that the values of A_1 and A_2 are scaled down by a factor of $e^{\beta^2\sigma_s^2/2}>1$. While a unique exchange rate is defined as a function of the fundamentals for the traditional (credible) target zone, a family of such curves is defined for the perceived soft target zone. The dynamics becomes what we can call a soft edge target zone. Figure 4 portrays the features of the exchange rates versus fundamentals of the models for σ_s = 0.0, σ_s = 0.005, and σ_s = 0.01.[16]

Several properties of the solution are apparent from Figure 4. First, the resulting dynamics of the exchange rate are again *S*-shaped. In the fully credible target zone solution (σ_s = 0), the exchange rate is stabilised at the edges [$\underline{s}=\ln(7.75)$ and $\overline{s}=\ln(7.85)$]. Second, for perceived soft edges (σ_s > 0), the resulting *S*-shaped curve is a monotonically increasing function of the standard deviation. The other way round, the moderating effect is the stronger, the better the reputation of the policymaker. Third, while all curves are almost identical near s = ln (7.80), the soft band curves become steeper than the hard band curve as the exchange rate approaches the edges. Hence, soft edge bands lead to weaker honeymoon effects and wider target zone ranges.

[16] We are well aware of the stylised nature of the model, despite our attempts to calibrate with realistic parameters. We do not claim empirical accuracy for the model but use it rather for qualitative features and predictions.

Figure 4: Soft Edge Target Zone Model



Note: Benchmark values are $\sigma = 0.03$, $\overline{s} = \ln(7.85)$, $\underline{s} = \ln(7.75)$, and $\lambda = 0.9$.

It makes sense to assume that the longer the exchange rate complies with the pre-announced band, the more the announcement is believed. As a result, the commitment grows stronger and more credible over time. Let us consider this dependence on the history of the exchange rate, by re-writing σ_s^2 as $\sigma_s^2 e^{-\eta t}$, where $\eta > 0$ denotes the credibility convergence speed.[17] In other words, the dynamics causing the effective width of the target zone evolve over time as long as $\overline{s} \geq s \geq \underline{s}$. By modelling the time-varying nature of the variance, we approximate the current configuration in Hong Kong which has secured exchange rate stability.

It is straightforward to verify that by taking expectations we obtain the following equations describing the dynamics of the exchange rate in the target zone:

(14)
$$E\left[s \middle| \overline{s}, \sigma_s^2 e^{-\eta t}\right] = E\left[f - A_1 e^{\beta f} + A_2 e^{-\beta f} \middle| \overline{f}, \sigma_s^2 e^{-\eta t}\right]$$
$$\Rightarrow \overline{s} = \overline{f} - \left(A_1 e^{\beta \overline{f}} - A_2 e^{-\beta \overline{f}}\right) e^{\beta^2 \sigma_s^2 e^{-\eta t}/2},$$

(15)
$$E\left[s \middle| \underline{s}, \sigma_s^2 e^{-\eta t}\right] = E\left[f - A_1 e^{\beta f} + A_2 e^{-\beta f} \middle| \underline{f}, \sigma_s^2 e^{-\eta t}\right]$$
$$\Rightarrow \underline{s} = \underline{f} - \left(A_1 e^{\beta \underline{f}} - A_2 e^{-\beta \underline{f}}\right) e^{\beta^2 \sigma_s^2 e^{-\eta t}/2}.$$

The associated smooth pasting conditions are given by

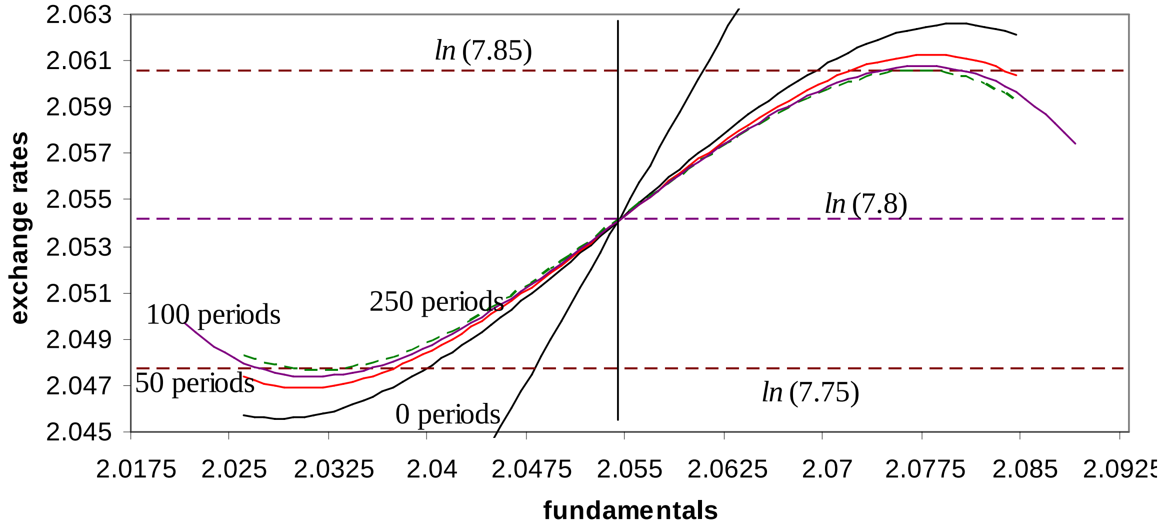
[17] Here we can conveniently assume that the time-dependent exogenous variances do not affect the Bellman equation (3), as they are exogenous to market participants.

(16) $$0 = 1 - A_1 \beta e^{\beta\left(\bar{f} + \beta\sigma_s^2 e^{-\eta t}/2\right)} - A_2 \beta e^{-\beta\left(\bar{f} - \beta\sigma_s^2 e^{-\eta t}/2\right)},$$

(17) $$0 = 1 - A_1 \beta e^{\beta\left(\underline{f} + \beta\sigma_s^2 e^{-\eta t}/2\right)} - A_2 \beta e^{-\beta\left(\underline{f} - \beta\sigma_s^2 e^{-\eta t}/2\right)},$$

where $\bar{f}$ and $\underline{f}$ are again the upper and lower values of the fundamentals corresponding to the upper and lower bounds of the target zone, respectively. A numerical solution is shown in Figure 5.

Figure 5: Exchange Rates as a Function of Fundamentals over Time



Note: Parameters of the model are $\eta = 4$. Curves are drawn for $t = 50$, $t = 100$ and $t = 250$ days.

The situation portrayed in Figure 5 resembles that in Figure 4. The dynamics cause the width of the band to be a varying function of time; over time, the central bank gains credibility among market participants.

The type of problem analysed above has room for further interactions between the exchange rate dynamics and perceived credibility. So far constant or deterministically declining σ_s^2 values have been assumed. To go beyond the previous exercises, we pursue the analysis further by augmenting the target zone framework with a bivariate Markov-switching model.[18] In other words, we model the vulnerability of target zone regimes to perceived future speculative attacks and possible self-

[18] An interesting alternative to our modelling approach with oscillating band widths over time has recently been presented by Dibeh (2006). In his modelling approach, the periodic motion is forced by a sinusoidal function.

fulfilling equilibria.[19] The implications of such regime-dependence on the target zone dynamics has not been investigated yet.

How do we introduce such dynamics into the target zone model? To make the perceived nonlinear swings in exchange rate pressure tractable, a stochastic process for σ_s^2 must be specified. We assume that two different regimes for σ_{si}^2 with $\sigma_{s1}^2 > \sigma_{s0}^2$ characterised by either episodes of exchange rate stability (i = 0) or periods of turbulence (i = 1) exist. The process is subject to discrete (sporadic) regime shifts governed by a two-regime Markov process with constant transition probabilities.[20] Formally, the probability of being in each state is determined by the transition equation

(18) prob[S_{1t} = 1|$S_{1,t\text{-}1}$ = 1] = 1- ϕ, prob[S_{0t} = 0|$S_{0,t\text{-}1}$ = 0] = 1-θ.

The simple and elegant structure of the Markov-switching framework makes it possible to derive simple and elegant results. The altered value-matching conditions for both states are as follows:

(19) $$\overline{s}_0 = \overline{f}_0 - \left(A_{10}e^{\beta \overline{f}_0} - A_{20}e^{-\beta \overline{f}_0}\right)e^{\beta^2 \sigma_{s0}^2/2} + \theta\left(\overline{s}_1 - \overline{s}_0\right),$$

(20) $$\underline{s}_0 = \underline{f}_0 - \left(A_{10}e^{\beta \underline{f}_0} - A_{20}e^{-\beta \underline{f}_0}\right)e^{\beta^2 \sigma_{s0}^2/2} + \theta\left(\underline{s}_1 - \underline{s}_0\right);$$

(21) $$\overline{s}_1 = \overline{f}_1 - \left(A_{11}e^{\beta \overline{f}_1} - A_{21}e^{-\beta \overline{f}_1}\right)e^{\beta^2 \sigma_{s1}^2/2} + \phi\left(\overline{s}_0 - \overline{s}_1\right),$$

(22) $$\underline{s}_1 = \underline{f}_1 - \left(A_{11}e^{\beta \underline{f}_1} - A_{21}e^{-\beta \underline{f}}\right)e^{\beta^2 \sigma_{s1}^2/2} + \phi\left(\underline{s}_0 - \underline{s}_1\right),$$

where $\overline{s}_0$ and $\overline{s}_1$ and $\underline{s}_0$ and $\underline{s}_1$ are the upper and lower exchanges rate bands, respectively. $\overline{f}_0$, $\underline{f}_0$, $\overline{f}_1$ and $\underline{f}_1$ are the corresponding fundamental thresholds for both bands in regime 0 and regime 1, and A_{10}, A_{20}, A_{11} and A_{21} are the constant terms for both regimes.

[19] Evidence for repetitive pressure on financial markets leading to a distinct cyclical motion and a new wave of exchange rate speculation has been provided by Westerhoff and Reitz (2003). They demonstrate that in the presence of chartists which use trends for trading in foreign exchange markets, currencies deviate from their fundamental value and follow a distinct cyclical behaviour. Furthermore, the focus upon market expectations implies that our modelling approach stands in the tradition of the second-generation currency crisis model literature.

[20] Hamilton (1990), Engel (1994), and Cheung and Erlandsson (2005) have popularized the Markov-switching toolkit in exchange rate economics by showing that the Markov switching model is a relevant statistical alternative to the classical martingale model for exchange rates.

Irrespective of the expectations of market participants, the central bank acts on the assumption of given thresholds, i.e. $\overline{s} = \overline{s}_0 = \overline{s}_1$ and $\underline{s} = \underline{s}_0 = \underline{s}_1$. In other words, the central bank tries to avoid the credibility cost of giving up the exchange rate. Hence, also the thresholds for the fundamental are the same for both regimes: $\overline{f} = \overline{f}_0 = \overline{f}_1$ and $\underline{f} = \underline{f}_0 = \underline{f}_1$. Given this assumption the solution can be written as

$$(23) \qquad \overline{s} = \overline{f} - \left(A_{10} e^{\beta \overline{f}} - A_{20} e^{-\beta \overline{f}} \right) e^{\beta^2 \sigma_{s0}^2 / 2},$$

$$(24) \qquad \underline{s} = \underline{f} - \left(A_{10} e^{\beta \underline{f}} - A_{20} e^{-\beta \underline{f}} \right) e^{\beta^2 \sigma_{s0}^2 / 2},$$

$$(25) \qquad \overline{s} = \overline{f} - \left(A_{11} e^{\beta \overline{f}} - A_{21} e^{-\beta \overline{f}} \right) e^{\beta^2 \sigma_{s1}^2 / 2},$$

$$(26) \qquad \underline{s} = \underline{f} - \left(A_{11} e^{\beta \underline{f}} - A_{21} e^{-\beta \underline{f}} \right) e^{\beta^2 \sigma_{s1}^2 / 2}.$$

After some simple calculations we get the regime-dependent dynamics of the exchange rate in the target zone

$$(27) \qquad s_0 = f - \left(A_{10} e^{\beta f} - A_{20} e^{-\beta f} \right) + \theta \left(s_1 - s_0 \right),$$

$$(28) \qquad s_1 = f - \left(A_{11} e^{\beta f} - A_{21} e^{-\beta f} \right) + \phi \left(s_0 - s_1 \right).$$

Finally, rearranging yields the following regime-dependent *S*-shaped functions:

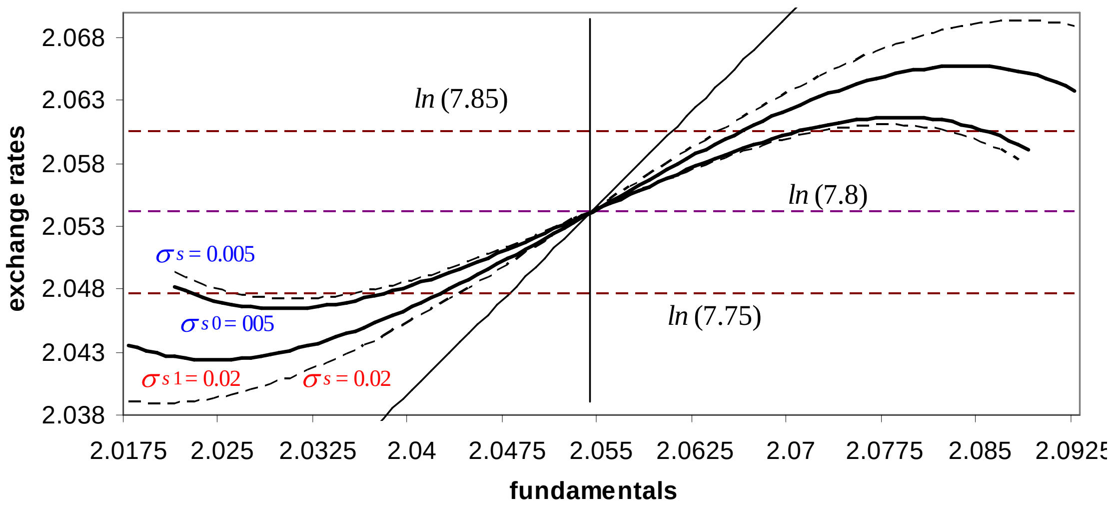
$$(29) \qquad s_0 = f - \left(A_{10} e^{\beta f} - A_{20} e^{-\beta f} \right) - \frac{\theta}{1+\theta+\phi} \left[\left(A_{11} e^{\beta f} - A_{21} e^{-\beta f} \right) - \left(A_{10} e^{\beta f} - A_{20} e^{-\beta f} \right) \right],$$

$$(30) \qquad s_1 = f - \left(A_{11} e^{\beta f} - A_{21} e^{-\beta f} \right) - \frac{\phi}{1+\theta+\phi} \left[\left(A_{10} e^{\beta f} - A_{20} e^{-\beta f} \right) - \left(A_{11} e^{\beta f} - A_{21} e^{-\beta f} \right) \right].$$

The last terms of the right-hand sides of equations (29) and (30) are precisely the market participants' expectations of future regime changes.

Figure 6 shows representations of the regime-dependent family of *S*-curves. The plots are generated by numerical solution of the differential equation system for various times *t*. The parameters are θ = 0.2 and ϕ = 0.67, respectively.

Figure 6: The Soft Edge Band Model With and Without Markov-Switching



Note: The dotted (solid) lines represent the dynamics of the exchange rate without (with) regime switching.

Three interesting results emerge. First, it is notable that the target zone remains a stabilising instrument even under regime-switching expectations as evidenced by the *S*-shaped dynamics. Second, the story is more subtle than has been acknowledged so far. Figure 6 indicates that in the Markovian version of the model the perceived width of the band oscillates. An intuitive explanation is the following. In the low-volatility regime 0, the expectation that a future regime change is more likely in a high-volatility regime explains why the perceived target zone range is increasing in comparison with the baseline model in Figure 4. This is interesting in as far as the central bank is assumed to maintain the band. Thus, perceived possible regime switches and speculative attacks may put the central bank under strain. Third, in the high-volatility regime 1 the perceived width of the band is decreasing in comparison with the baseline model in Figure 4. This moderating effect results from the fact that the system will switch back to the low-volatility regime in the future. As seen by market participants, this makes a widening of the bands less likely.

4. Summary and Conclusions

Over the last decade, central banks have implemented new breeds of target zone exchange rate regimes. In Hong Kong, a symmetric band forming upper and lower limits for HKD fluctuations around the central parity was adopted in May 2005, as an integral part of the currency board regime.

An interesting feature of Hong Kong´s currency board arrangement is the important achievement of speculative peace and credibility after the turbulent year 2003.[21] Thus, the formal shift to a very narrow target zone of ±0.6 percent in May 2005 has so far helped in ending, mitigating, and preventing excessive speculative pressure.[22]

On the occasion of the tenth anniversary of Hong Kong´s return to China, *Time* (2007) magazine ran a cover story on Hong Kong entitled "Sunshine with Clouds, 1997 - 2007". In close analogy to the title, we summarise the recent performance of the HKD – USD peg as "Mostly Sunshine with Occasional Clouds, 2005 – 2009". Going from specifics to generalisations, one can perhaps now conclude that narrow bands may experience an unanticipated comeback in conjunction with currency board arrangements.[23]

This experience is diametrically opposed to the ERM experience in the 1990s. Speculative attacks on the French Franc led to the so-called Brussels Compromise in August 1993, which led to much wider fluctuation bands of ±15 percent ("soft buffers"). By dropping the commitment to keep the exchange rate within a narrow band, ERM central banks have thwarted one-way bets and speculative spurts [Batolini and Prati (1993, 1997) and Labhard and Wyplosz (1996)].[24]

For our modelling framework we have demonstrated that a narrow target zone may remain a stabilising instrument even under noisy expectations. Clearly, this sort of analysis is only a first step. Nevertheless, the analysis may lead to enhanced understanding of the great resilience to speculative pressure of Hong Kong´s tight ±0.6 percent target zone since May 2005. The modelling work may also shed a new light on the economics of target zone regimes.[25]

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[21] However, in October 2007 strong upward pressure on the HKD put the future of the currency board again in the spotlight, and HKMA was forced to intervene in the foreign exchange markets to prevent the rate from breaching the narrow band. This represented the first such intervention by the HKMA in more than two years.

[22] Cautionary note: the current stability may be coincidental and caused by the current undervaluation of the HKD. In spring 2009 the HKD turned out to be 52 percent undervalued by the *Economist* Big Max burger gauge. The Big Max index is a light-hearted PPP measure of currencies (see *The Economist*, 18 July 2009, p. 66).

[23] One should, however, qualify this statement by saying that according to the results for the augmented Markov-switching version of the target zone model in Figure 6, the exchange rate may still be vulnerable to future speculative attacks.

[24] Actually, the ERM policy approach since August 1993 may be interpreted as a managed-float regime in which central banks had intervened with increasing intensity as the exchange rates departed from central parities, but without commitment to an explicit narrow band.

[25] Whereas this paper focuses on exchange rate dynamics, other aspects, such as optimal band width are left to future research. Some papers have endogenised exchange rate policy by deriving the width of the band as a rational choice of an optimising central bank. For example, Cukierman et al. (1994) model the choice of band width as a choice between flexibility in responding to external shocks and commitment to less devaluation and inflation.

Appendix: Derivation of Equations (10) and (11)

Despite its conceptual simplicity, the model is rather tedious to solve. Below we give the details of the solution. The derivation of (10) and (11) can be explained as follows. Taking expectations of equation (5) with $N\left(\overline{f}, \sigma_s^2\right)$, we obtain

(A1) $$E\left[s \middle| \overline{s}, \sigma_s^2\right] = \overline{s} = \overline{f} + \frac{1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} \left(-A_1 e^{\beta f} + A_2 e^{-\beta f}\right) e^{-\frac{\left(f-\overline{f}\right)^2}{2\sigma_s^2}} df \ .$$

Removing all constant parameters from the integral yields

(A2) $$\overline{s} = \overline{f} - \frac{A_1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{\beta f} e^{-\frac{\left(f-\overline{f}\right)^2}{2\sigma_s^2}} df + \frac{A_2}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{-\beta f} e^{-\frac{\left(f-\overline{f}\right)^2}{\sigma_s^2}} df \ .$$

The next step is to obtain expressions for the integrals on the right-hand side of (A2). By expanding $\left(f - \overline{f}\right)^2$ and collecting terms related to *f*, the first integral is given by

(A3) $$\frac{A_1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{\beta f} e^{-\frac{\left(f-\overline{f}\right)^2}{2\sigma_s^2}} df = \frac{A_1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{-\frac{f^2 - 2\overline{f}f + \overline{f}^2 - 2\beta\sigma_s^2 f}{2\sigma_s^2}} df \ .$$

Completing a square of $\left(f - \left(\overline{f} + \beta\sigma_s^2\right)\right)^2$ gives

(A4) $$\begin{aligned} &\frac{A_1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{-\frac{f^2 - 2f\left(\overline{f}+\beta\sigma_s^2\right) + \left(\overline{f}+\beta\sigma_s^2\right)^2 - \left(\overline{f}+\beta\sigma_s^2\right)^2 + \overline{f}^2}{2\sigma_s^2}} df \\ &= \frac{A_1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{-\frac{\left(f-\left(\overline{f}+\beta\sigma_s^2\right)\right)^2 - \left(2\overline{f}\beta\sigma_s^2 + \beta^2\sigma_s^4\right)}{2\sigma_s^2}} df = A_1 e^{\beta\left(\overline{f}+\beta\sigma_s^2/2\right)} \frac{1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{-\frac{\left(f-\left(\overline{f}+\beta\sigma_s^2\right)\right)^2}{2\sigma_s^2}} df. \end{aligned}$$

Note that the integral $\frac{1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{-\frac{\left(f-\left(\overline{f}+\beta\sigma_s^2\right)\right)^2}{2\sigma_s^2}} df$ is the expectation of normal distribution with mean $\overline{f} + \beta\sigma_s^2$ and variance σ_s^2, and equals one. Thus the preceding expression simplifies to

(A5) $$\frac{A_1}{\sqrt{2\pi\sigma_s^2}} \int_{-\infty}^{\infty} e^{-\beta f} e^{-\frac{\left(f-\overline{f}\right)^2}{2\sigma_s^2}} df = A_1 e^{\beta\left(\overline{f}+\beta\sigma_s^2/2\right)}.$$

Based on the same logic, it is straightforward to demonstrate that the second integral is of the form

(A6)

$$\frac{A_2}{\sqrt{2\pi\sigma_s^2}}\int_{-\infty}^{\infty} e^{-\beta f} e^{-\frac{\left(f-\overline{f}\right)^2}{2\sigma_s^2}}df = -\frac{A_2}{\sqrt{2\pi\sigma_s^2}}\int_{-\infty}^{\infty} e^{-\frac{f^2-2\overline{f}f+\overline{f}^2+2\beta\sigma_s^2 f}{2\sigma_s^2}}df$$

$$= \frac{A_2}{\sqrt{2\pi\sigma_s^2}}\int_{-\infty}^{\infty} e^{-\frac{f^2-2v\left(\overline{f}-\beta\sigma_s^2\right)+\left(\overline{f}-\beta\sigma_s^2\right)^2-\left(\overline{f}-\beta\sigma_s^2\right)^2+\overline{f}^2}{2\sigma_s^2}}df$$

$$= \frac{A_2}{\sqrt{2\pi\sigma_s^2}}\int_{-\infty}^{\infty} e^{-\frac{\left(f-\left(\overline{f}-\beta\sigma_s^2\right)\right)^2-\left(-2\overline{f}\beta\sigma_s^2+\beta^2\sigma_s^4\right)}{2\sigma_s^2}}df$$

$$= A_2 e^{-\beta\left(\overline{f}-\beta\sigma_s^2/2\right)}\frac{1}{\sqrt{2\pi\sigma_s^2}}\int_{-\infty}^{\infty} e^{-\frac{\left(f-\left(\overline{f}-\beta\sigma_s^2\right)\right)^2}{2\sigma_s^2}}df = A_2 e^{-\beta\left(\overline{f}-\beta\sigma_s^2/2\right)}$$

After plugging in (A5) - (A6) and rearranging, we finally obtain equation (10) in the text:

(A7) $$E\left[s\middle|\overline{s},\sigma_s^2\right] = \overline{s} = \overline{f} - A_1 e^{\beta\left(\overline{f}+\beta\sigma_s^2/2\right)} + A_2 e^{-\beta\left(\overline{f}-\beta\sigma_s^2/2\right)}$$

In the same vein, equation (11) can be obtained. Taking expectations of equation (11) in the text with $N\left(\underline{f},\sigma_s^2\right)$ yields

(A8)

$$E\left[s\middle|\underline{s},\sigma_s^2\right] = \underline{s} = \underline{f} + \int_{-\infty}^{\infty}\left(-A_1 e^{\beta f} + A_2 e^{-\beta f}\right)e^{-\frac{\left(f-\underline{f}\right)^2}{2\sigma_s^2}}df$$

$$= \underline{f} - \frac{A_1}{\sqrt{2\pi\sigma_s^2}}\int_{-\infty}^{\infty} e^{\beta f} e^{-\frac{\left(f-\underline{f}\right)^2}{2\sigma_s^2}}df + \frac{A_2}{\sqrt{2\pi\sigma_s^2}}\int_{-\infty}^{\infty} e^{-\beta f} e^{-\frac{\left(f-\underline{f}\right)^2}{\sigma_s^2}}df$$

Following the above simplifying procedure, we obtain

(A9) $$\underline{s} = \underline{f} - A_1 e^{\beta\left(\underline{f}+\beta\sigma_s^2/2\right)} + A_2 e^{-\beta\left(\underline{f}-\beta\sigma_s^2/2\right)},$$

which concludes the derivation.

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